IX*—THE TOTALITY OF FACTS

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ABSTRACT Wittgenstein, in the *Tractatus*, conceives the world as 'the totality of facts'. Type-stratification threatens that conception: the totality of facts is an obvious example of an illegitimate totality. Wittgenstein's notion of truth-operation evidently has some role to play in avoiding that threat, allowing propositions, and so facts, to constitute a single type. The paper seeks to explain that role in a way that integrates the 'philosophical' and 'technical' pressures on the notion of an operation.

I

The first sentences of Wittgenstein's *Tractatus* announce:

> The world is everything that is the case.
> The world is the totality of facts, not of things. (TLP 1, 1.1)[1]

But can we understand that '*everything*'? Is there any such totality?

Logic, or more guardedly our understanding of the general force of logical principles, seems to demand that the answer is 'Yes'. 'Whatever is, is' surely involves just that understanding of 'everything'. On the other hand, the logical background from which Wittgenstein set out to write the *Tractatus* contains familiar reasons for holding that the answer must be 'No'. The very next sentence echoes one such ground:

> The world is determined by the facts, and by these being *all* the facts. (TLP 1.11)

Is 'these being *all* the facts' intended to register one of the facts that determines the world? If so, the remark runs straight into the vicious circle principle,

1. References to Wittgenstein's works are given using abbreviations explained at the end of the paper.

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> [which] may be stated as follows: 'Whatever involves *all* of a collection must not be one of the collection'; or conversely: 'If, provided a certain collection had a total, it would have members only definable in terms of that total, then the said collection has no total'... [where] By saying that a set has 'no total', we mean, primarily, that no significant statement can be made about 'all its members'.[2]

The point is surely one we are supposed to notice.

To address the tension locally, by analytically defusing the apparent circle in remark 1.11, would be a mistake. It's true that Wittgenstein's subsequent treatment of generality (TLP 5.52 ff.) implicitly rejects Russell's argument for irreducibly general facts; and that if, in line with that treatment, we were to imagine a 'fully analysed' version of 'these being all the facts'—as, roughly, a vast conjunction containing every elementary proposition or its negation—we would conclude that it involves (entails) each of the facts while remaining innocently on the same level as them. But to respond in that way would be just to attack the flag, ignoring what Wittgenstein is using it to signal. The case for the answer 'No' doesn't rest on this particular instance. As Russell says, consideration of any general principle of logic would lead, in the setting of the vicious circle principle, to the conclusion that 'Propositions... must be a set having no total'.[3]

It would also be a mistake to try to undermine the answer 'Yes' through the thought that, for Wittgenstein, *essential* truth, rather than *general* truth, is what characterizes logic (TLP 6.1231). In the first place, to hold that their maximal generality is not enough to mark off logical principles from others is not to deny that they have this generality. In the second, while the *Tractatus* identifies the propositions of logic with tautologies (TLP 6.1) rather than, as in his earliest thoughts, generalizations of them (CL p. 59), Wittgenstein's account of the use of logical principles (TLP 6.126, cf. NM p. 112) demands that what one discerns in recognizing a given tautology as such is something quite general. That the essential validity of logic should also be general validity underlies the demand that there be a general form of proposition (TLP 4.5). So the answer 'Yes' seems non-negotiable.

2. Whitehead and Russell, *Principia Mathematica*, I, p. 37.

3. ibid.

The general form of proposition is specified in the *Tractatus* through the notion of an *operation*, the notion that distinguishes Wittgenstein's account of the logical constants from those of Frege and Russell. To reach a stable answer to our question we therefore need to understand how that account allows Wittgenstein to avoid the threat to the generality of logic posed by Russell's adoption of a theory of types. Wittgenstein's pronouncements on the logical constants range from the seemingly trivial—'That $\vee$, $\supset$, etc., are not relations in the sense of right and left, etc., is obvious' (TLP 5.42)—to the bizarre—'... there are NO *logical* constants' (CL p. 14). I think it is only by displaying their connections with this issue that we can understand these remarks as cohesive and coherently motivated.

Explaining that will take up pretty well the whole paper. But I should say a word about background motivations. Some writers on Wittgenstein's early work, most notably perhaps Gordon Baker,[4] insist that the point of his logical pronouncements is exclusively 'philosophical', and so neither motivated by nor testable against issues in 'technical' logic. Some others, notably Michael Potter,[5] compensate by placing the logical innovations of the *Tractatus* in a history of the technical development of the logicist approach to arithmetic. It seems plain to me that Wittgenstein himself drew no distinction between 'philosophical' and 'technical' problems in logic, so that to understand his early work we have to find ways to integrate what may seem to us to be two different kinds of concern. In the course of the discussion I will mention two forms that the threat to the generality of logic might take, which are readily enough counted one 'philosophical' and the other 'technical'. Displaying the cohesion of Wittgenstein's response to them is thus a small step in what I think is a necessary direction.

II

Let's begin with the seemingly trivial remark quoted above. Who needs to be told that *or* and *only if* are not like *right* and *left*? There are two likely answers, both right. The ancestor of the

4. *Wittgenstein, Frege and The Vienna Circle*; see especially p. 73.

5. *Reason's Nearest Kin*, ch. 6.

Tractatus passage occurs in the *Notes on Logic* (NL p. 101), where the point is said to be 'obvious to the plain man'. Wittgenstein dictated these notes in October 1913 to summarize his progress for Russell, and in many parts of the notes Russell himself is the direct target. But two months later, before Christmas 1913, Wittgenstein went to present his results in discussion with Frege and—according to an obviously dubious report from Goodstein—'wiped the floor' with him.[6]

Frege is the more obvious target, since he did straightforwardly maintain that the truth-functional constants were first-level relational expressions, that is, incomplete expressions with gaps for singular terms. In *Grundgesetze* all complete expressions, simple and complex singular terms and propositions, are of the same logical category. Since the logical category of all other expressions is fixed relatively to complete expressions, there is no logical difference between an expression that takes two terms to make a proposition and an expression that takes two propositions to make a bigger proposition. No one since Frege has followed him in this, so an argument targeted narrowly against that Fregean view would not be terribly interesting. Fortunately, Wittgenstein's arguments have broader aims.

One of them runs thus:

> Logical indefinables cannot be predicates or relations, because propositions, owing to sense, cannot have predicates or relations. (NL p. 99)

The argument turns on two central ideas, each instancing the general claim that 'Symbols are not what they seem to be' (NL p. 98). The first is that a predicate, what symbolizes a property, is not just a predicate sign such as the letter '*F*', but consists in '*F*''s standing in some conventionally significant connection with a name. Correspondingly, a relational symbol, what symbolizes a relation, is not merely a sign like '*R*', but consists in '*R*''s occurring between two names of things thereby said to stand in the relation. The second, connected idea is that 'Propositions are not names' (NL p. 98). More fully, a symbol representing that such-and-such a fact obtains is itself a fact (NL p. 97). 'Only facts can express a sense' (NL p. 105). So the proposition '*aRb*' consists in

6. McGuinness, *Wittgenstein, A Life: Young Ludwig*, pp. 186–7, 190–1.

the fact that the name '*a*' stands in a relation indicated by '*R*' to the name '*b*'.

If both points are allowed the inference is immediate. For a proposition, say '*aRb*', to 'have a predicate', say '*F*', would be for '*F*' to stand to the left of the fact that '*R*' is between '*a*' and '*b*'. But that is just nonsense (cf. NM p. 116).

That argument turns, as I said, on two ideas. In the case of the first, I just want to register a corollary of it before putting it on one side. If what means a certain property is not just the sign '*F*', but a particular kind of occurrence of that sign in a propositional context, then it is perfectly possible that the sign '*F*' should occur in some way in a proposition without meaning that property. To some other kind of use we might have assigned a quite different meaning; and to many uses that might be made of it we will have assigned no meaning at all. Mere sameness of letter tells us nothing about sameness of meaning.

> It can never express the common characteristic of two objects that we designate them by the same name but by two different ways of designation, for, since names are arbitrary, we might also choose different names, and where then would be the common element in the designations? (NL p. 97; cf. TLP 3.322)

Wittgenstein's second idea, that a proposition is a fact, certainly blocks Frege's assimilation of (e.g.) the negation sign to a predicate. But its way of doing that seems like overkill. It indeed makes no sense to suppose that the negation sign might stand to a propositional-fact in the kind of relation that a predicate sign stands to a name when it means a property. But that just seems to instance a more general point: we have no notion at all how the word 'not' could be put into *any* kind of significant connection with a propositional-fact so as to signify its negation. Frege's model goes, but only, it seems, because we now have *no* model of how a proposition and a negation operator can combine to form a proposition.

That may well strike us as an obvious ground for complaint. But right or not it is exactly what Wittgenstein intended. What at first looks like a narrow criticism of Frege is, because of that consequence, seen to be directed against *any* account according to which a truth-functionally complex proposition such as '*p*&*q*' has the propositions '*p*' and '*q*' as constituents—in any sense of 'constituent' which is even remotely like that in which the names '*a*' and '*b*' are constituents of '*aRb*'. Thus:

> There are *internal* relations between one proposition and another; but a proposition cannot have to another *the* internal relation which a *name* has to the proposition of which it is a constituent, and which ought to be meant by saying that it 'occurs' in it. In this sense one proposition can't 'occur' in another. (NM p. 116)

In this, 'name' should, I think, be understood with the full breadth of Frege's usage in *Grundgesetze*, to mean any expression replaceable by a variable to yield what we might call an open sentence, or what Russell called a propositional function, or what Wittgenstein called simply a function. So read the claim is: there are no functions whose arguments are propositions. The so-called *Grundgedanke* (TLP 4.0312), that there are no logical constants, is a somewhat narrower consequence of this claim.

III

Before connecting the claim with Russell it will be as well to recall how it is worked out in the *Tractatus*'s account of truth-operations. (The terminology of 'operations' does not occur in the early notes, and some of the connected theory did not appear until *Prototractatus*; but all of the points I'll draw on were already there in *Notes on Logic*.)

According to that account, the result of applying a truth-operation to '*p*' is a truth-function of '*p*', that is, its truth-value is a function of the truth-value of '*p*' (TLP 5.234). The sense of a truth-function of '*p*' is a function of the sense of '*p*' (TLP 5.2341). But a proposition that is a truth-function of '*p*' not itself a function of '*p*'. This is because the occurrence of a truth-operator in a complex proposition characterizes neither its sense (TLP 5.25) nor its form (TLP 5.241).

In support of that last claim, Wittgenstein points to features of the behaviour of truth-operators: they can cancel out (TLP 5.253), or 'vanish' (TLP 5.254); and they can be amalgamated, in that the result of successive truth-operations 'is also the result of *one* truth-operation' (TLP 5.3). The general idea is that truth-operations have an effect on their bases, but leave no trace of themselves in their results.

The point is clearest in Wittgenstein's preferred TF-notation, which uses the truth-table of a proposition as its symbol, so that what we might write as '$\neg(p\supset\neg q)$' or as '$p\&q$' appear indifferently as '(TFFF)(p,q)'. The negation operation then clearly appears as a way of turning one proposition into another, thus:

$$\begin{array}{l}(\mathrm{T\ F\ F\ F})\ (p,q)\\ \ \ |\ \ |\ \ |\ \ |\\ (\mathrm{F\ T\ T\ T})\ (p,q)\end{array}$$

In this notation it is plain that none of the operations by which we might have constructed the proposition '(FTTT)(p,q)' characterizes its form or its sense. We imagined reaching it by negating '(TFFF)(p,q)', but there is nothing essentially 'negative' about the result. Negation is characteristic only of the relation between two propositions, never of any proposition itself (TLP 5.241).

It is easy to forget that what the proposition '(FTTT)(p,q)' emerges from by application of truth-operations are not 'p' and 'q' but '(TF)(p)' and '(TF)(q)'; and that *these* do not figure as constituents of that proposition is again manifest in the notation. Speaking loosely, one might say that '(FTTT)(p,q)' is Wittgenstein's equivalent of Sheffer's '$p|q$', but if equivalence implies corresponding behaviour that is wrong. '$p|q$' is viewed as having 'p' and 'q' as its constituents, and as itself a constituent of further propositions, '$p|(p|q)$' for instance. But the proposition '(FTTT)(p,q)' *cannot* be embedded in a more complex proposition. It is essentially a *complete* proposition. It is in that way that the notation gives recognizable shape to the contention of the Moore notes, that one proposition cannot occur in another.

More generally, the TF-notation of the *Tractatus* realizes all of Wittgenstein's early claims about the logical constants. But then it was designed for just that purpose. It can help clarify the claims, but cannot do much to motivate them. Conversely, Wittgenstein's observations about the alleged behaviour of the ordinary operators have, I think we should admit, little force in themselves, independently of the conception embodied in this notation. We need a further point of leverage to display the value of that conception.

IV

That point emerges, as I said, in connection with the threat to the generality of logic deriving from Russell's adoption of the theory of types. I'll mention two forms it takes, the first more familiar than the second.

(i) The familiar idea is that, if the logical constants are just some among the things there are, then logic has no business dealing with them.

> A reason against [logical constants] is the generality of logic: logic cannot treat a special set of things. (NL p. 98)
> Not only must logic not deal with [particular] things, but just as little with relations and predicates. (NL p. 98)

Sometimes this thought is presented, appealingly, as a matter of maintaining the dignity of logic, as it was later by Russell:

> Pure logic... aims at being true... in all possible worlds, not only in this higgledy-piggledy job-lot of a world in which chance has imprisoned us. There is a certain lordliness which the logician should preserve: he must not condescend to derive arguments from the things he sees about him.[7]

But the difficulty needn't be one of arriving at a suitably lofty conception of logic's concerns; it may just be a problem of arriving at any consistent conception.

When he wrote the *Principles* in 1903 Russell thought of logic as a science of maximal generality, definable as 'the class of propositions containing only variables and logical constants'.[8] He wrote:

> So long as any term in our proposition can be turned into a variable, our proposition can be generalized; and so long as this is possible, it is the business of mathematics [and so of logic] to do it.[9]

Together these claims imply that the logical constants cannot be 'turned into a variable', but why not?

7. *Introduction to Mathematical Philosophy*, p. 192.
8. *Principles of Mathematics*, p. 9.
9. *Principles of Mathematics*, p. 7.

In the *Principles* the answer turns on the fact that 'variables have an absolutely unrestricted field: any conceivable entity may be substituted for any one of our variables'.[10] A variable thus has no particular symbolic shape to it, so turning *everything* in a proposition into a variable would give us just a shapeless mush. But by the time of *Principia* Russell had been forced by the paradoxes to abandon that conception of the variable, so that a variable now ranges only over things of the same logical type as the constant it replaces. With that change Russell lost his reason for holding that the place of a logical constant is not accessible to a variable (and Russell certainly intends in *Principia* that truth-functions fall within the range of function variables; see, e.g., his gloss on 9.14*). But he had not given up the idea that logic must generalize wherever it can. So by his own conception of the subject the basic laws of *Principia* have no business being in the book at all. The need to avoid that inconsistency was part of what motivated Wittgenstein's reconception of the logical constants:

> Logical constants can't be made into variables: because in them *what* symbolizes is *not* the same; all symbols for which a variable can be substituted symbolize in the *same* way. (NM p. 114)

(ii) The second issue reintroduces the corollary I put to one side near the beginning, namely that mere sameness of sign tells us nothing about sameness of meaning. That figures essentially in Wittgenstein's resolution of the Russell paradox, first presented in *Notes on Logic* (NL p. 96) but more fully stated in the *Tractatus*.

> A function cannot be its own argument, because the functional sign already contains the prototype of its argument and it cannot contain itself.
>
> If, for example, we suppose that the function $F(fx)$ could be its own argument, then there would be a proposition '$F(F(fx))$', and in this the outer function F and the inner function F must have different meanings; for the inner has the form $\varphi(fx)$, and the outer the form $\psi(\varphi(fx))$. Common to both functions is only the letter 'F', which by itself signifies nothing...
>
> Herewith Russell's paradox vanishes. (TLP 3.333)

The appearance of paradox arises when we assign to the inner function '$F(fx)$' the meaning that fx does not apply to itself, and

10. ibid.

assume that by that stipulation '$F(F(fx)$' makes the paradoxical assertion that not applying to itself does not apply to itself. The appearance dissolves when we realize that our stipulation about the inner 'F' is silent about the outer 'F', which for all we've said might mean anything or nothing.

The question is: why does the same reasoning not apply to a truth-functional operator, such as negation?[11] Consider '$\neg\neg p$'. Suppose p is atomic, and that we have somehow fixed the significance of applying '$\neg$' to it. Will it not be right to argue, as before, that for all we have so far fixed the outer '$\neg$' might mean anything or nothing?

> The outer function $\neg$ and the inner function $\neg$ must have different meanings; for the inner has the form $\varphi(p)$ and the outer the form $\psi(\varphi(p))$. Common to both is only the sign '$\neg$', which by itself signifies nothing.

Michael Potter spells out an argument to that effect.

> If... negation is thought of as a function which takes a proposition p as argument and gives the proposition $\neg p$ as result, it presupposes a range of propositions which can serve as arguments to it and by the vicious circle principle the result of applying the function cannot be in the original range of arguments. It follows... that in the proposition $\neg\neg p$ the two occurrences of the sign '$\neg$' are different symbols. [12]

A directly applicable formulation of the vicious circle principle is the following:

> ... there must be no propositions, of the form φx, in which x has a value which involves $\hat{\varphi}x$.[13]

Reading $\neg$ for φ, it follows from this that $\neg$ cannot be univocally reapplied to yield $\neg\neg p$ unless $\neg p$ is not 'a value which involves [$\neg$]'.[14]

11. The question is raised by Ishiguro in 'Wittgenstein and the Theory of Types', at p. 54. In contrast to Ishiguro, I think Wittgenstein's answer to it has to be sharply distinguished from any that Russell could offer.

12. *Reason's Nearest Kin*, ch. 6.

13. *Principia Mathematica* I, p. 40.

14. On Russell's own account, should he have accepted that $\neg p$ involves $\neg$? Russell maintains: 'in any particular case... a value of a function does not presuppose the function... e.g. the proposition "Socrates is human" can be perfectly apprehended without regarding it as a value of the function "x is human"' (*Principia* I, p. 40; cf. pp. 54–5). However things stand with Socrates and humanity, the idea that $\neg p$ should be apprehended 'without regarding it as' the negation of p seems hopeless.

Russell's hierarchy is founded on elementary propositions, a single type including atomic propositions *and* truth-functions of atomic propositions. So one would expect

It might seem clear enough already how this reapplication of the paradox-resolving reasoning is to be blocked. If we can have an understanding of how negation works on which $\neg p$ does not involve $\neg$, that will remove any obstacle from the vicious circle principle to reapplying the same operation to it to yield $\neg\neg p$. But we saw that on Wittgenstein's account of the constants the proposition $\neg p$—i.e. the proposition (FT)(p)—does *not* involve $\neg$. On that account there is, as I put it, nothing especially 'negative' about a proposition that we happen to have arrived at by applying the negation operation. So the problem is resolved.

In fact, I think that is too quick. The apparent parallel between the two pieces of reasoning does have to be undermined, and Wittgenstein's understanding of the truth-operators does have a role in undermining it—but, as I'll explain shortly, not quite the role just sketched. Before entering into the detail needed to explain that, however, we should stop to ask what would follow if the parallel reasoning went through.

The consequence would be, I think, that Russell's logic would have to abandon its last claim to formulate general principles. In accepting type theory Russell had already surrendered the ideal of absolute generality maintained in the *Principles*. Within the restrictions of his theory of types there can be no generalizations over *all* propositions, or *all* properties of an individual, and so on. In consequence, most of those things in *Principia* which look like asserted propositions are no such thing. For example,

$$((Fx \supset_x Gx)\ \&\ (Gx \supset_x Hx)) \supset (Fx \supset_x Hx)$$

looks like the principle of Barbara, that for any properties F, G, H, if everything F is G and everything G is H, then everything F is H. But given the restrictions of type theory, there is no such thing as *the* principle of Barbara; instead there is an infinite hierarchy of principles in which F, G, and H are taken to range over properties of increasing order. So if the above formula appeared in *Principia*, it would not be a single assertion, but a stand-in for infinitely many distinct assertions. Such a stand-in is a 'type-ambiguous' assertion. The impression that we understand such an assertion is largely owed, I think, to the implicit assumption that at least its logical framework carries a unitary significance.

to find in that connection, if anywhere, a justification of his claim (*9.1331, *Principia* I, p. 133) that truth-functions do not push one up the type hierarchy. Unfortunately, what one finds instead is an equivocation on 'elementary' (*Principia* I, pp. 44–6).

The principal effect of the parallel reasoning would be to expose that assumption, showing that there is no logical commonality between the different disambiguations of the assertion.[15] Further, even within a single unambiguous reading of the above formula the main implication sign must be a different symbol from the implication signs that occur in its scope. The operators can bear no consistent reading even within the simplest laws of propositional logic.

V

So, how is that consequence to be avoided? To answer that, we need to re-examine Wittgenstein's resolution of the Russell paradox. As noted, a necessary premise of the resolution is that mere sameness of sign tells us nothing about sameness of meaning. This makes possible the resolution's claim that the inner and outer '*F*'s in '*F*(*F*(*fx*)' are different symbols, but of itself does not entail it. One needs some reason for maintaining that those '*F*'s *do* symbolize in different ways. It is to meet this need that the reconstruction of the argument sketched in the previous section contains an appeal to the vicious circle principle as providing that reason. But I think Wittgenstein's argument contains no such appeal.

Taking Wittgenstein's notation at face value, the variable '*fx*' marking the argument-place of the inner '*F*' in '*F*(*F*(*fx*)' is a first-level functional variable. So relying on no more than the simple hierarchy generated by the Fregean conception of incompleteness, and endorsed in Wittgenstein's claim that a function 'contains the prototype of its argument' (TLP 3.333), one arrives at the conclusion that the inner '*F*' symbolizes a second-level

15. So far the reapplication of Wittgenstein's argument just makes vivid what is already implicit in Russell. Explaining the type-ambiguous version of LEM, he says: 'we may give to *p* a value of any order, and then give to the negation and disjunction involved those meanings appropriate to that order' (*Principia* I, p. 129). But what *are* those 'appropriate meanings'? If Russell had held that the meanings of the logical operators are fixed by the axioms or by the inference rules governing them, then there would be a response open to him: the meaning fixing stipulations are themselves (to be understood as) type-ambiguous. But that was not his view. From the *Principles* in 1903 through to *Theory of Knowledge* in 1913, he maintained that understanding of logical notions is grounded in a kind of acquaintance. But then understanding the type-ambiguous $p \vee \neg p$ calls for infinitely many distinct acts (or episodes?) of acquaintance.

function and therefore the outer 'F' a function of third level. So 'F' is two different symbols in its two occurrences.[16]

If that *were* Wittgenstein's reasoning then it would be enough, as Frege immediately recognized,[17] to stop the property version of the Russell paradox; and it would have no tendency to generalize to the two '$\neg$'s in '$\neg\neg p$', since the type of expression they are applied to is, by Fregean standards, the same.

It is very natural to assume, however, that this cannot be all there is to Wittgenstein's reasoning. He presents himself as resolving Russell's paradox generally, and the paradox also comes in a propositional version, based on the notion of a 'class-assertion', a proposition asserting the truth of all propositions belonging to a certain class or satisfying a certain function.[18] If we take it that the propositional paradox is within the intended scope of Wittgenstein's resolution, then we must also take it to be incidental that Wittgenstein writes '$F(F(fx)$' rather than '$F(F(p)$', and on that assumption we cannot reach the conclusion that the two 'F's symbolize differently on merely Fregean grounds. At this point appeal to something like the vicious circle principle would be unavoidable: if F cannot be univocally iterated in '$F(F(p)$' the reason lies in something more fine-grained that the (Fregean) type of its arguments, and that something must surely be the Russellian basis of the ramified hierarchy.

That Wittgenstein's paradox resolution must be elaborated in this way is, as I said, a very natural assumption. Nonetheless I think it is a mistake. The essential point to note is that the resolution's move onto Russellian ground would not be needed at all if Wittgenstein had other grounds for rejecting the propositional

16. In this paragraph my account of the argument simply follows Ishiguro's in 'Wittgenstein and the Theory of Types'; I depart from Ishiguro below, in holding that the argument given here needs no supplementation.

17. *Philosophical and Mathematical Correspondence*, p. 132.

18. The propositional version of the paradox is presented in Appendix B of the *Principles* (p. 527), and is a focus of the Frege-Russell correspondence. For present purposes it is enough to explain the idea of a 'class-assertion' that generates it. For any class of propositions m there is—or seems to be—a proposition, $\forall p.\, p \in m \supset p$. For instance, if m has as members all propositions asserted by the policeman, then $\forall p.\, p \in m \supset p$ amounts to 'Everything the policeman said is true'. So we have—it seems—a mapping from classes of propositions m to propositions $\forall p.\, p \in m \supset p$. That puts us in obvious contradiction with Cantor's theorem, that the subclasses of a class (here, the class of all propositions) cannot be put into correspondence with the members of it. Wittgenstein's claim that propositions 'cannot have predicates' rules out the formulation $p \in m$, and therewith the very notion of a class-assertion.

version of the paradox—if, in particular, he had grounds for denying quite generally that there are such functions, taking propositions as their arguments, as are appealed to in the notion of a class-assertion. But we saw in Section II that Wittgenstein was led to just that general conclusion by his understanding of a proposition as a representing fact. That general claim, of which the *Grundgedanke* is a consequence, thus allows Wittgenstein to restrict the scope of his paradox resolution to the property version of Russell's paradox. It can then go through on purely Fregean grounds, without reliance on the vicious circle principle. And because of that, the parallel reasoning that proves so threatening to Russell's position no longer *needs* to be blocked: it doesn't even get started.

The argument for the *Grundgedanke* expounded in Section II occurs in the immediate context of, and shares premisses with, Wittgenstein's first formulation of his resolution of Russell's paradox (NL p. 96). This supports my suggestion that we should see the two arguments as having operated in concert: the *Grundgedanke* argument restricts the paradox to a version resolvable by the simple, Fregean hierarchy. Understood in this way Wittgenstein's response to the paradox is much more directly a model for Ramsey's. As in Ramsey, the only stratification called for is of propositional functions by their argument places; propositions are of a single type. Also as in Ramsey, this simplification is made possible by a division of labour, with the type structure called upon to resolve only some of what Russell had understood to be a single kind of paradoxes. (On the other hand, Ramsey's redeployment of the ramified structure to resolve the semantic paradoxes has, so far as I can see, no precedent in Wittgenstein.)

VI

So long as Wittgenstein's paradox-resolution is thought to involve reliance on the vicious circle principle, the prospect of parallel reasoning will dictate an understanding of truth-operations that puts them beyond the principle's reach. To meet that need Michael Potter recommends that we understand Wittgensteinian operations as, in effect, mappings on a domain of *senses* of propositions, where the sense of a proposition involves no more than is displayed in its truth-table. Senses so understood have no internal structure: the result of an operation for a given basis will involve neither the operation nor the basis; and so there can be no objection from the vicious circle principle to reapplying to the result that same operation. More generally, we have an assurance against paradoxical explosion, in the prior and determinate conception of the domain of these operations as comprising, to speak loosely, all possible fillings of an all-inclusive truth-table.

Potter's recommended understanding of operations certainly meets the case in hand, but only, I think, at the cost of distorting or neglecting other aspects of the notion. An operation is:

> ... the expression of a relation between the structures of its result and its bases. (TLP 5.22)

Thus,

> The first place in which an operation can occur is where a proposition arises from another in a logically significant way. (TLP 5.233)

Neither point holds in general for mappings from senses to senses; for instance, they do not hold of a mapping that takes the sense of one atomic proposition to that of another. More generally, these other aspects of the notion of an operation will be in place only if it is not just *incidental* that an operation is specified as a way of transforming one proposition into another; but that *will* appear incidental, if operations are fundamentally to be understood by reference to a prior and independent conception of their domain.

So long as the vicious circle principle is in play, iteration of an operation Ω to yield $\Omega\Omega p$ is blocked if Ωp 'involves' or presupposes Ω. Potter ensures iterability by recommending instead an understanding of operations on which Ω presupposes Ωp. But what Wittgenstein was aiming at was an understanding on which, for operations, in contrast to functions, presupposition is mutual:

> Logical functions all presuppose one another. Just as we can see $\neg p$ has no sense, if p has none; so we can also say p has none if $\neg p$ has none. The case is quite different with φa and a; since here

> *a* has a meaning independently of φa, though φa presupposes it. (NM p. 118)

Anscombe insightfully pointed to a danger for readings of the *Tractatus*, that they will effectively attribute to Wittgenstein a composite theory of the proposition: a picture theory of atomic propositions *and* a theory of truth-functions.[19] Supposing that truth-operations have to be understood by reference to an already-in-place domain of senses is, I think, one way of succumbing to that danger, while stressing the two-way dependence maintained in the Moore notes is part of avoiding it.

VII

What, then, is the upshot? So far as concerns the different interpretive approaches I mentioned at the beginning, we have—stepping back now from details—a happy outcome. Baker insists that Wittgenstein's thought about the logical constants rests on a supposed insight in the metaphysics of symbolism, that a proposition is a fact. Potter's more mathematical approach presents it as a response to pressures from the theory of types. Both are substantially right; my only broad criticism of either approach is that it needs to make room for the other.

And where do we stand with my original question: is there a totality of facts? I have considered only one kind of reason for saying 'No', and in deflecting it have tried to accommodate the most basic reason for saying 'Yes'. Wittgenstein's account of the logical constants provides for propositions to form a single logical type; it thus frees from the suspicion cast on it by the threatened parallel reasoning our intuitive understanding of the general force of logical principles.

Similarly freed—and, in the context of the *Tractatus*, much more importantly freed—from that same source of suspicion are the following.

> The general form of truth-function is $[\bar{p}, \bar{\xi}, N(\bar{\xi})]$.
> This is the general form of proposition. (TLP 6)

> To give the essence of the proposition means to give the essence of all description, therefore the essence of the world. (TLP 5.4711)

19. *Introduction to Wittgenstein's Tractatus*, pp. 25–6.

That essence could not have been given, in the way Wittgenstein purports to in TLP 6, if the parallel reasoning had ruled out a single, univocal reading of 'N'. Providing for such a reading reopens the prospect that everything there is can be embraced by the single language I understand, whose unitary and transparent perspective I occupy, and through which I conceive how things are. Russell's opposed view carried a threat of disintegration: on the one hand of the world, as comprising *everything* that is the case; on the other of the self, or as one might also say, of the perspective on that world which my language provides.

That makes it tempting to answer my original question with an outright 'Yes'. But there are reasons for hesitating that I haven't considered here at all. For instance, if we hear 'totality' as indicating a 'completed' or 'determinate' totality, then a positive answer would present the idea of a 'fully analysed' language as an idea of something actual, rather than, as I would sooner understand it, as an ideal pointer. And even assuming the notion of 'totality', there is the quite different question, untouched by anything I've said here, why the world should be a totality of *facts* rather than of *things*. So, on the broadest understanding of the original question, the only conclusion reached here is: Maybe.[20]

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20. Talks variously overlapping with this one were given during 1998–9 at the universities of Wisconsin Milwaukee, Cambridge, Sheffield and London, and at a graduate study weekend on the Isle of Raasay. Thanks to members of those audiences, especially Fraser McBride, for questions and criticisms, and to Michael Potter for many discussions of these and related issues.

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